Functions for portfolio selection

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https://CRAN.R-project.org/package=NMOF

A main topic of Gilli et al. [2019] is non-standard portfolio-selection models; see Chapters 12–14. Nevertheless, the NMOF package also offers several functions that help with standard portfolio models, i.e. models that can be solved with traditional optimisation techniques such as quadratic programming.

1 Minimum-variance portfolios

The function minvar computes the minimum-variance portfolio for a given variance–covariance matrix, subject to holding-size constraints. As example data, the variable var contains a small variance–covariance matrix, computed from daily returns of five German stocks. The data are taken from http://enricoschumann.net/data/gilli_accuracy.html ; the code to build the matrix is in the source file of this vignette.

minvar

```
> var
           CBK.DE    VOW.DE   CON.DE   LIN.DE  MUV2.DE
CBK.DE   0.000988 -1.80e-05 3.69e-04 2.08e-04 2.63e-04
VOW.DE  -0.000018  1.72e-03 8.57e-05 2.15e-05 2.84e-05
CON.DE   0.000369  8.57e-05 7.59e-04 1.94e-04 1.89e-04
LIN.DE   0.000208  2.15e-05 1.94e-04 2.66e-04 1.33e-04
MUV2.DE  0.000263  2.84e-05 1.89e-04 1.33e-04 2.59e-04
```

An example call, with minimum and maximum holding sizes specified.

```
> minvar(var, wmin = 0, wmax = 0.5)
[1] 6.94e-18 9.25e-02 4.69e-05 4.45e-01 4.62e-01
attr(,"variance")
[1] 0.000182
```

The function returns the portfolio weights with an attribute variance that provides the variance of this portfolio. The holding size constraints can also be specified as vectors, with different values for different assets.

```
> minvar(var,
         wmin = c(0.1, 0, 0, 0, 0), ## enforce at least 10% weight in CBK.DE
         wmax = 0.5)
```

Use Inf to switch off weight constraints.

```
> minvar(var, wmin = -Inf, wmax = Inf)   ## no bounds
> minvar(var, wmin = -Inf, wmax = 0.45)  ## no lower bounds
> minvar(var, wmin =  0.1, wmax = Inf)   ## no upper bounds
```

The function also supports group constraints:

```
> ## group 1 consists of asset 1 only,   and must have weight [0.25,0.30]
> ## group 2 consists of assets 4 and 5, and must have weight [0.10,0.20]
> minvar(var, wmin = 0, wmax = 0.40,
         groups = list(1, 4:5),
         groups.wmin = c(0.25, 0.1),
         groups.wmax = c(0.30, 0.2))
[1] 0.250 0.217 0.333 0.149 0.051
attr(,"variance")
[1] 0.000357
```

Alternatively, group constraints can be specified through group names instead of positions.

```
> ## group A consists of asset 1 only,   and must have weight [0.25,0.30]
> ## group B consists of assets 4 and 5, and must have weight [0.10,0.20]
> minvar(var, wmin = 0, wmax = 0.40,
         groups = c("A", "none", "none", "B", "B"),
         groups.wmin = c(A = 0.25, B = 0.1),
         groups.wmax = c(A = 0.30, B = 0.2))
```

```
[1] 0.250 0.217 0.333 0.149 0.051
attr(,"variance")
[1] 0.000357
```

2 Mean–variance efficient portfolios and frontiers

The function `mvPortfolio` computes a mean–variance-efficient portfolio for a given variance–covariance matrix and mean-return assumption, subject to holding-size constraints. We make up some data for four assets, with a constant correlation of 0.5. `mvPortfolio`

```
> vols <- c(0.10, 0.15, 0.20, 0.22)  ## expected vols
> m    <- c(0.06, 0.12, 0.09, 0.07)  ## expected mean returns
> const_cor <- function(rho, na) {
      C <- array(rho, dim = c(na, na))
      diag(C) <- 1
      C
  }
> var <- diag(vols) %*% const_cor(0.5, length(vols)) %*% diag(vols)
```

One way to compute a mean–variance-efficient portfolio is by requiring a minimum return.

```
> mvPortfolio(m, var, min.return = 0.08, wmax = 1)
```

```
[1] 0.667 0.333 0.000 0.000
```

```
> mvPortfolio(m, var, min.return = 0.10, wmax = 1)
```

```
[1] 3.33e-01 6.67e-01 0.00e+00 3.10e-18
```

```
> mvPortfolio(m, var, min.return = 0.12, wmax = 1)
```

```
[1] -1.11e-16  1.00e+00 -5.55e-17  2.64e-17
```

Alternatively, we may specify a trade-off between return and variance and minimise

$$-\lambda \mathtt{m}'w + \frac{1}{2}(1-\lambda)w'\mathtt{var}\,w,$$

in which w are the weights. If λ is a vector of length 2, then the function minimises

$$-\lambda_1 \mathtt{m}'w + \frac{1}{2}\lambda_2 w'\mathtt{var}\,w.$$

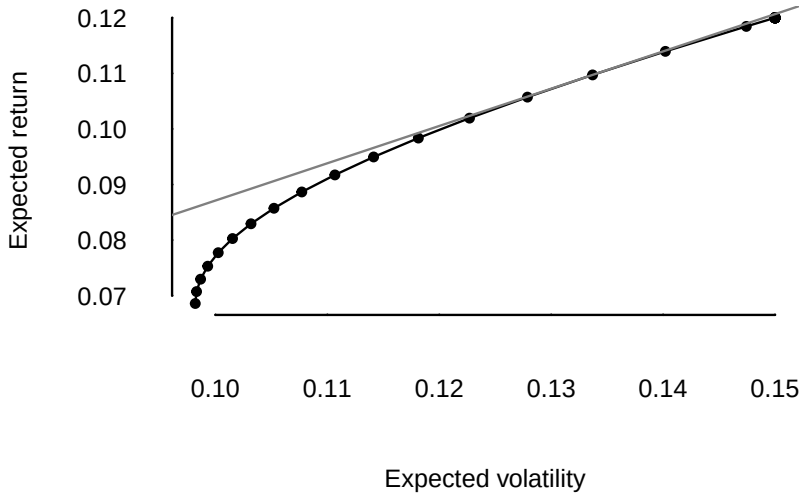
The function `mvFrontier` traces out a whole frontier of mean–variance efficient portfolios. (But see the discussion on frontiers in Chapter 14 of Gilli et al., 2019.) `mvFrontier`

```
> if (requireNamespace("quadprog")) {
      wmin <- 0
      wmax <- 1
      p1 <- mvFrontier(m, var, wmin = wmin, wmax = wmax, n = 50)

      ## with a 'risk-free' asset rf
```

```
    rf <- 0.02
    p2 <- mvFrontier(m, var, wmin = wmin, wmax = wmax, n = 50, rf = rf)

    par(las = 1, bty = "n", tck = 0.001, ps = 8)
    plot(p1$volatility, p1$return, pch = 19, cex = 0.5, type = "o",
         xlab = "Expected volatility",
         ylab = "Expected return")
    lines(p2$volatility, p2$return, col = grey(0.5))
    abline(v = 0, h = rf)
} else
    plot(1)
```



3 Return-based tracking portfolios

Function `trackingPortfolio` computes a portfolio that is close to another portfolio in the mean-square/ variance sense. The function to be minimised is determined by argument `objective`: supported are `variance` (the default) or `sum.of.squares`. `trackingPortfolio`

```
> ns <- 120
> R <- randomReturns(na = 1 + 10,  ## first asset is the benchmark
                     ns = ns,
                     sd = 0.03,
                     mean = 0.005,
                     rho = 0.7)
> var <- cov(R)
> trackingPortfolio(var, wmax = 0.4)
 [1] 9.87e-18 5.87e-02 7.99e-03 1.53e-01 0.00e+00 2.39e-01
 [7] 9.89e-02 1.69e-01 2.47e-01 2.72e-02
```

4 Minimum-Expected-Shortfall portfolios

The function `minCVaR` computes a portfolio that minimises conditional Value-at-Risk; its default method `minCVaR`

is the LP approach described in Rockafellar and Uryasev [2000]. See *Minimising Conditional Value-at-Risk (CVaR)* (`http://enricoschumann.net/notes/minimising-conditional-var.html`) for more details

```
> ns <- 5000  ## number of scenarios
> na <- 20    ## nunber of assets
> R <- randomReturns(na, ns, sd = 0.01, rho = 0.5)
> if (requireNamespace("Rglpk")) { ## example requires "Rglpk" package
      sol <- minCVaR(R, q = 0.1)
  } else
      message("Package ", sQuote("Rglpk"), " not available")
```

References

Manfred Gilli, Dietmar Maringer, and Enrico Schumann. *Numerical Methods and Optimization in Finance*. Elsevier/Academic Press, 2nd edition, 2019. URL `http://enricoschumann.net/NMOF`.

R. Tyrrell Rockafellar and Stanislav Uryasev. Optimization of conditional value-at-risk. *Journal of Risk*, 2 (3):21–41, 2000. doi: 10.21314/JOR.2000.038.