Before Markowitz, there was nothing.

Harry Markowitz

- portfolios matter, not single stocks: portfolio selection, rather than stock selection
- statistical properties of portfolios derived from single stocks

Harry Markowitz

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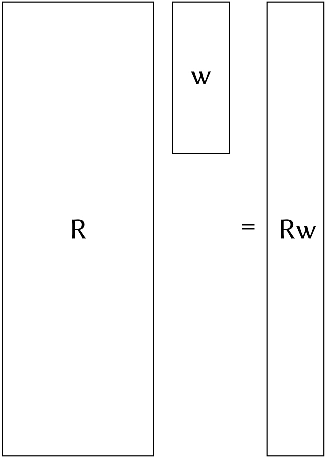
Harry Markowitz

- impact of long-only
- impact of correlations

Zero risk?



R returns, size $n_{scenarios} \times n_{assets}$

w portfolio weights

Rw portfolio returns

Zero risk?

$$m = \frac{1}{n_{\mathrm{S}}} \iota' R$$

$$\frac{1}{n_{\mathrm{S}}} R' R = \mathrm{Cov}(R) + m m'$$

Risk and reward

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characterise any portfolio by risk and reward

a portfolio is efficient if, for a level of risk, there is no portfolio with higher expected return (equivalently: if, for a level of return, there is no portfolio with less risk)

Risk and reward

Markowitz (1952): reward is expected portfolio return

$$\sum_{i=1}^{n_A} \mu_i w_i = \mu' w$$

and risk is portfolio-return variance

$$\sum_{i=1}^{n_A} \sum_{j=1}^{n_A} w_i w_j \sigma_{ij} = w' \Sigma w$$

Constructing efficient frontiers

$$\max_{w} \; \mu' w - \gamma w' \Sigma w$$

with constraints

$$\begin{aligned} w &\geq 0 \\ w'\iota &= 1 \end{aligned}$$

Quadratic Programming (QP)

in R, with `solve.QP` (package quadprog)

$$\min_{b} \quad -d'b + \frac{1}{2}b'Qb$$

subject to

$$A'b \geq b_0$$

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QP – objective function

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QP – objective function

$$-d'b + \frac{1}{2}b'Qb$$

make substitutions

$$d = -\mu$$
$$Q = -2\gamma\Sigma$$

and we get

$$\mu'w - \gamma w'\Sigma w$$

QP – constraints $A'b \geq b_0$

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$$A' = \begin{bmatrix} 1 & 1 & \dots & 1 \\ -1 & & & \\ & -1 & & \\ & & \ddots & \\ & & & -1 \\ 1 & & & \\ & 1 & & \\ & & \ddots & \\ & & & 1 \end{bmatrix} = \begin{bmatrix} \iota_{n_A} \\ -I_{n_A} \\ I_{n_A} \end{bmatrix} \quad \text{and} \quad b_0 = \begin{bmatrix} 1 \\ -w_1^{\max} \\ -w_2^{\max} \\ \vdots \\ -w_{n_A}^{\max} \\ w_1^{\min} \\ w_2^{\min} \\ \vdots \\ w_{n_A}^{\min} \end{bmatrix}$$

$$\iota_{n_A} = \underbrace{[1, 1, 1, \dots]'}_{n_A}$$

QP – constraints

```
> na <- 3
> rbind(1, -diag(na), diag(na))
```

```
     [,1] [,2] [,3]
[1,]    1    1    1
[2,]   -1    0    0
[3,]    0   -1    0
[4,]    0    0   -1
[5,]    1    0    0
[6,]    0    1    0
[7,]    0    0    1
```

QP – constraints

```
> require("Matrix")
> na <- 3
> Matrix(rbind(1, -diag(na), diag(na)))
```

```
7 x 3 sparse Matrix of class "dgCMatrix"

[1,]  1  1  1
[2,] -1  .  .
[3,]  . -1  .
[4,]  .  . -1
[5,]  1  .  .
[6,]  .  1  .
[7,]  .  .  1
```

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References

Gilli, Manfred, Dietmar Maringer, and Enrico Schumann (2011). *Numerical Methods and Optimization in Finance.* Elsevier/Academic Press. URL: http://nmof.net.

Markowitz, Harry M. (1952). "Portfolio Selection". In: *Journal of Finance* 7.1, pp. 77–91.