A Note on Volatility Pumping

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1 Introduction

Volatility pumping is a trading strategy which, in essence, tries to exploit the fact the even though the correct measure to gauge the performance of a financial asset is the geometric mean return, its expectation at any point in time is its arithmetic mean return, which is always equal to or higher than the geometric mean. (For a much better introduction, see Luenberger (1998).)

To illustrate this point, assume two assets, one which stays constant in value (cash), whereas the other either rises by u, or goes down by $d = 1/u$ (both with probability .5). Assume $u = 1.2$, that is we have a rather volatile asset. Figure 1 shows several sample paths of this asset. (The price in period 0 equals 1.)

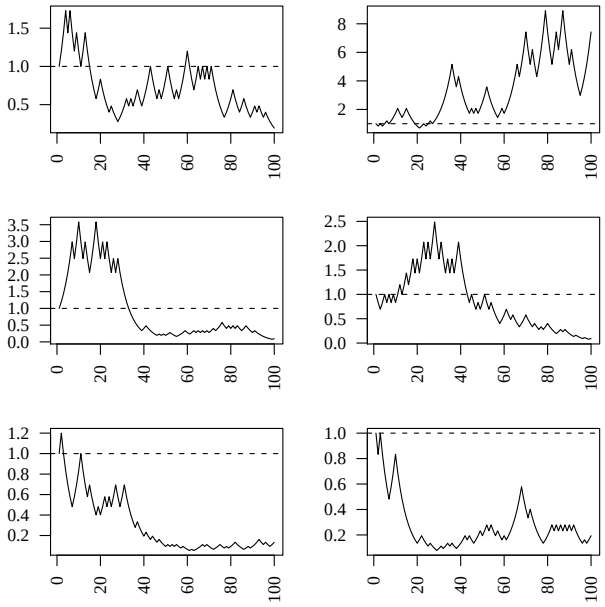

Figure 1: Sample series for risky asset.

The average geometric return of the risky asset is

$$d^{\frac{1}{2}} u^{\frac{1}{2}} = 1$$

However, the expected return in any period is the arithmetic mean, given by

$$\frac{1}{2}(d-1) + \frac{1}{2}(u-1) = .0\bar{3},$$

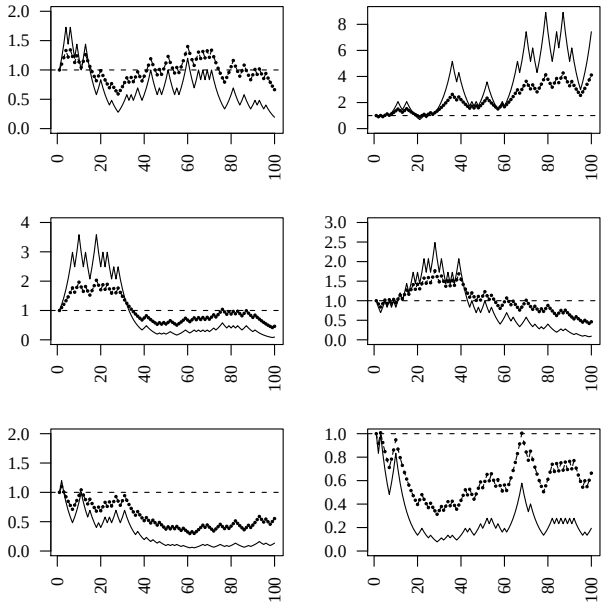

Figure 2: Sample series for risky asset with cash (50-50 portfolio). The dotted line represents the portfolio, the solid line the asset.

which is greater than zero. To exploit this, one can (in theory, of course, costlessly) shift one's wealth back and forth between the riskless asset and the risky one, thus increasing one's exposure to the risky asset when is has gone down, and decreasing it when it has risen. This resembles much the idea of 'buying low and selling high'. Unfortunately, it is also akin to the infamous 'martingale strategy', by which a gambler doubles the stakes after each loss so to get even with just one lucky trial.

Figure 2 shows how this strategy can work out in 'practice' when rebalancing is costless. Here 50% of the portfolio are invested into the risky asset, the rest is kept as cash. There are some important caveats: First, the returns are assumed to be independent. Dependence (say, correlation) is also important in the cross-section. In general, volatility pumping can be applied not just to one asset and cash, but to many risky assets simultaneously (in theory, this works much better than with cash). Volatility pumping will not work nicely, though, when applied to correlated assets which move up or down in lockstep. Second,

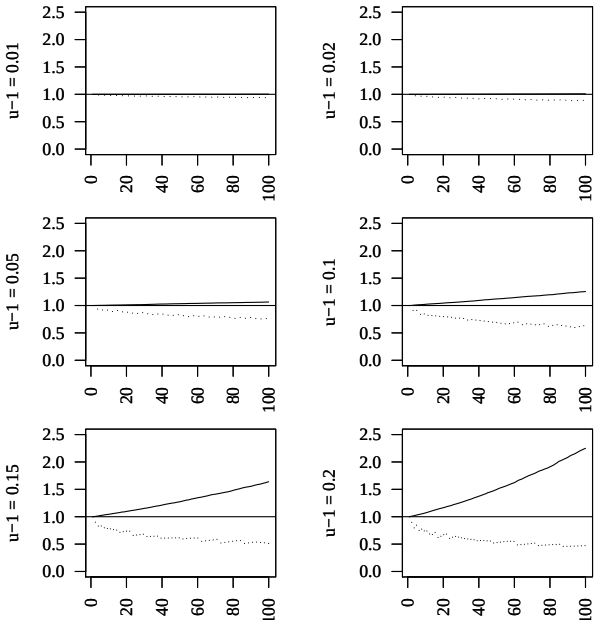

Figure 3: Mean path of 5,000 simulations of rebalancing strategy (solid line). The dotted line represents the 10th quantile to give an indication for the risk involved.

the higher the volatility, the better. For low volatility assets, the results may be less impressive. Third, all results rely on features which are certainly valid 'in the long run', but this does not necessarily imply desirable return properties of this strategy in finite samples.

2 Implications for trading strategies

Volatility pumping may be also applicable to trading strategies. In fact, trading strategies on several assets may be less correlated than these assets themselves. Simulations indicate that at least for only one risky asset and cash, volatility must be very high to produce reasonable results (see figure 3). Thus, to produce profits, a considerable amount of risk must be borne. ...

Further research has to be done on rebalanced portfolios of several risky assets (or trading strategies).

References

Luenberger, David G. (1998). *Investment Science*. Oxford University Press. Oxford.